Ellipses and Circles in Perspective

To help understand a circle in perspective, look first at a plan-view of a circle circumscribed by a square (as illustrated opposite page, top). Notice that the centre of the circle coincides with the centre of the square and that the circle touches the square at the mid-point of each side. It should therefore be a simple matter to transpose these conditions onto a square drawn in perspective.

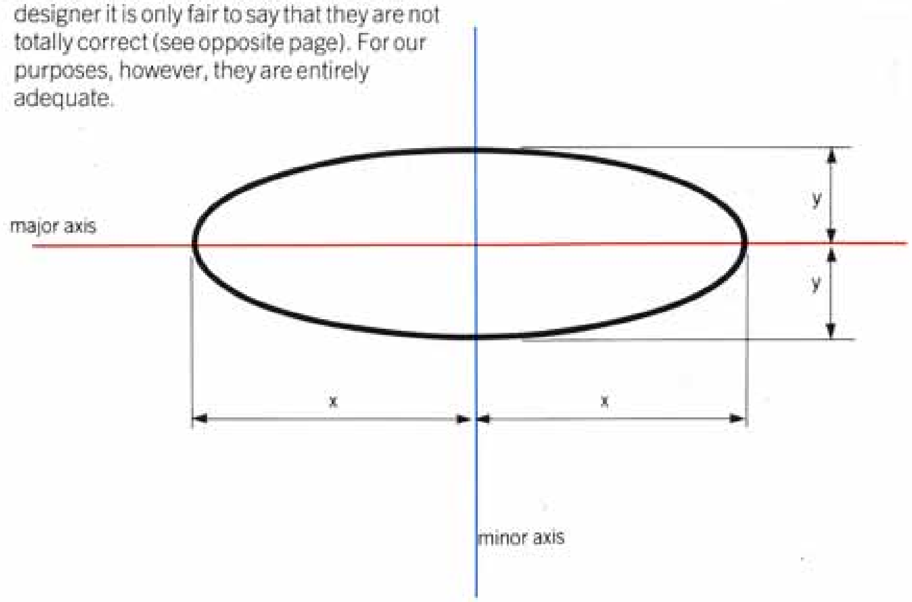
It can be shown mathematically that a circle in perspective is a true geometric ellipse, so it will also help to look at a true ellipse to see whether there is any help to be had from this end. An ellipse has a major and a minor axis at right angles to each other, it is also symmetrical about both axes so that each half of each axis is the same length. When an ellipse is viewed as a circle in perspective, the axis of rotation of that circle coincides with the minor axis.

This is the most common mistake made by students when first attempting to draw circles in perspective (except, strangely, those parallel to the ground like cups), because regardless of the view of the circle they nearly always draw the ellipse with the major axis vertical. If you are in any doubt about this, try drawing a range of perspective cubes above, below and on the horizon. If you then draw in an ellipse on one of the vertical faces, only on the horizon is the ellipse upright, and the more the cube is above or below the horizon the more the ellipse is leaned over. If the circle is parallel to the ground, it can be seen that its axis of rotation will be vertically downwards, and therefore the minor axis of the ellipse will also be vertically downwards, with the major axis at right angles to it and therefore parallel with the horizon. This is usually the easiest way to draw ellipses so, if you have trouble drawing freehand ellipses, turn the paper so that the minor axis is vertical on the paper and the major axis horizontal; it is also helpful to tick off lightly the intended distances along the axis.

Rules for drawing circles in perspective are therefore:

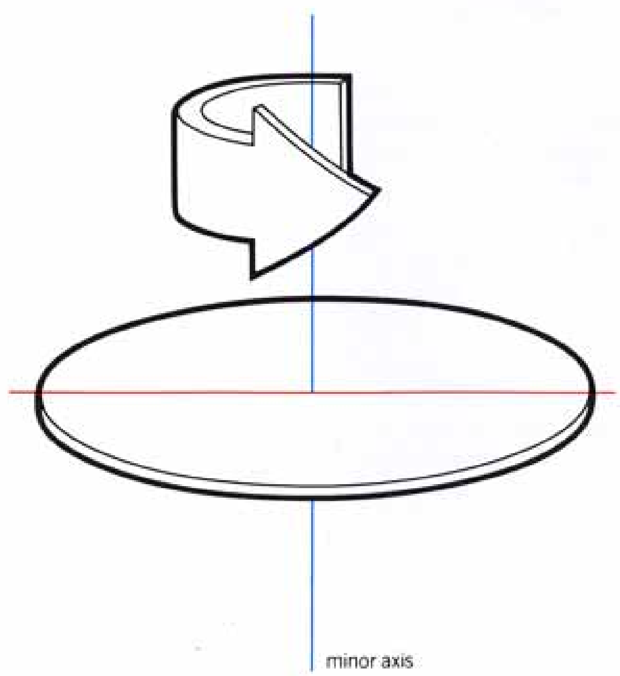
1. A circle in perspective is a true geometric ellipse.
2. A circle in perspective will touch its circumscribing square at the mid-point of each side.
3. A circle in perspective and its circumscribing square will share the same centre.
4. The ellipse will be geometrically symmetrical about its axis.
5. The axis of rotation of the circle will coincide with the minor axis of the ellipse.

Although these are important rules for the designer it is only fair to say that they are not totally correct (see opposite page). For our purposes, however, they are entirely adequate.



Above: Properties of ellipses
A geometric ellipse has a major and minor axis and is symmetrical about these axes (x=x and y=y).

Below: Axis of rotation
When an ellipse is viewed as a circle in perspective, the axis of rotation of that circle coincides with the minor axis. In other words, if the circle you are trying to draw was made of cardboard and you could spin it, then its axle, which of course must pass through the circle's centre and at right angles to it, will coincide with the minor axis of the ellipse.



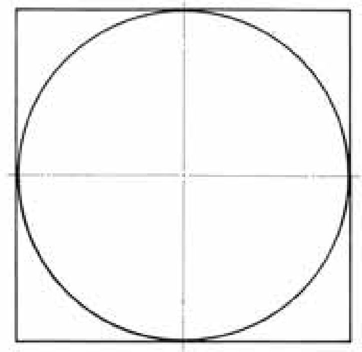
Right: Anomaly

1. As with the cube, look first at an elevation of the circle and its circumscribing square. The centre of the circle and the centre of the square coincide, and the circle touches the sides of the square at their mid-points.

2. This information can be transferred to the perspective view. However, as a student it seemed to me that a circle in perspective could not possibly be a true ellipse as can be shown thus:
Construct a square in perspective with its centre on the horizon. Next, divide it in half vertically by dropping a vertical line through the centre. (It will already be divided horizontally by the horizon.) As we have seen, our perspective circle will touch the circumscribing square at the mid-points of each side and will share the same perspective centre. Now, because of the effects of diminishing distance, that half of the square nearest to the vanishing point (x^1) must be smaller than that half further away from it (x). Equally, therefore, that half of the ellipse nearer the vanishing point must be smaller than that half nearer to you the viewer. Therefore it is not symmetrical and so cannot be a true geometric ellipse. The anomaly here is that, while a circle in perspective is indeed a true ellipse with its minor axis coinciding with the axis of rotation, the geometric centre of the ellipse does not coincide with the centre of the square *but is in fact shifted slightly towards the viewer.*

3. A schematic diagram may help you understand why this is so. Draw a circle with your eyepoint some way from it. Next, draw visual rays from this point, which are tangential to the circle, and then connect these tangent points together. What you actually see when looking at a circle in perspective is this chord, and the further away your eyepoint the nearer this centre gets to the true one.

4 and 5. For the same reason, it can be seen that the ellipses used to draw concentric circles in perspective cannot share the same geometric centre but will be offset slightly one from another. For all practical purposes this anomaly can be ignored because the offset is so small. Only those who are involved in super accurate measured construction and who will, in any case, be using a proper system need worry. Certainly your client will probably care little for the niceties of perspective theory!



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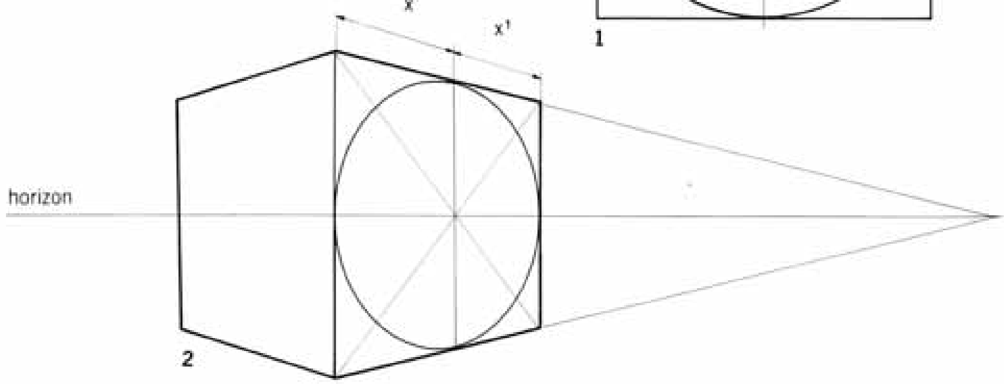

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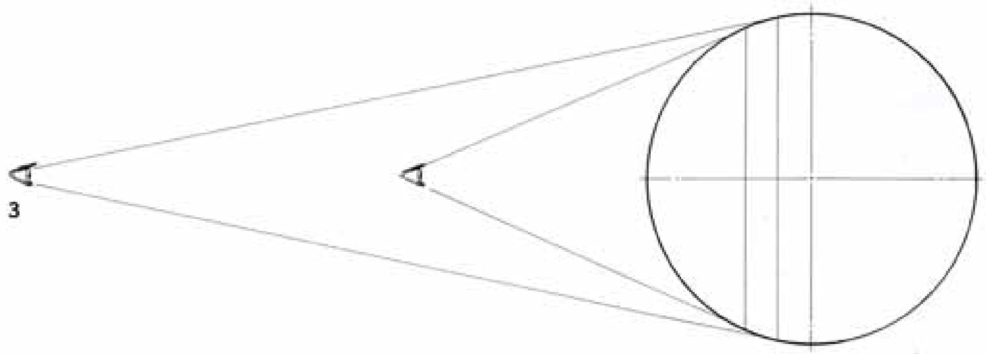

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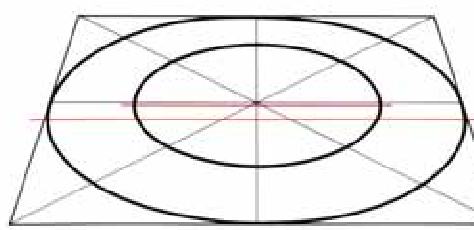

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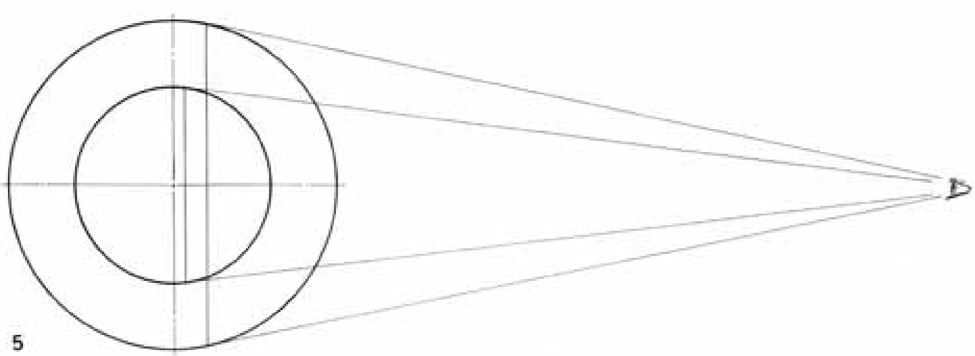

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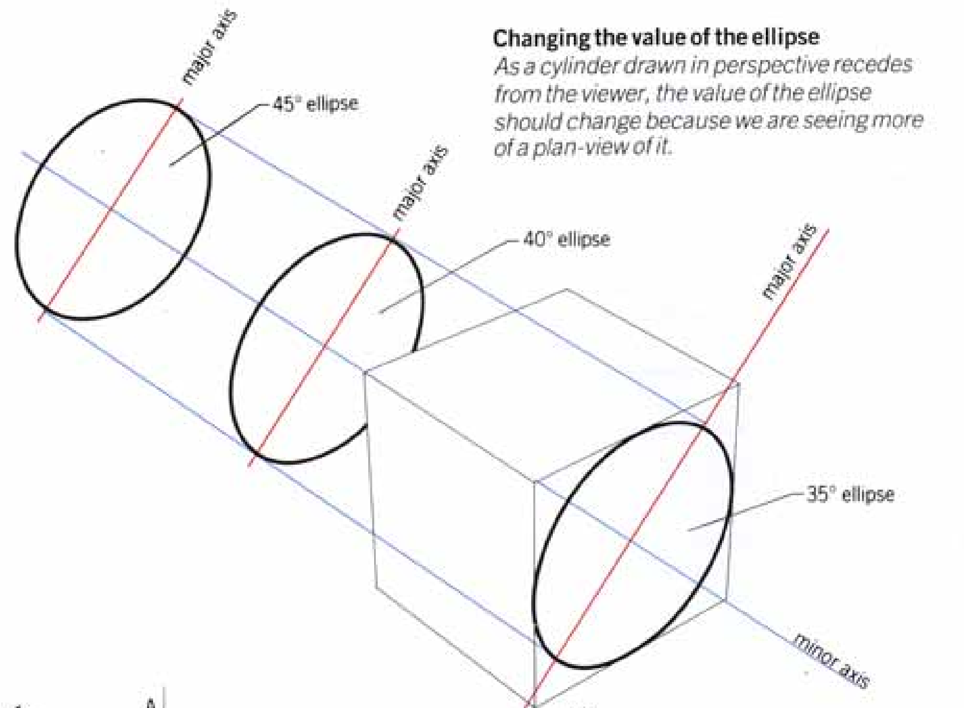
Ellipse guides/templates

Most designers will keep a set of ellipse guides simply because it is very hard to draw crisp ellipses freehand or even using French curves. Nevertheless, as with all perspective, do practise drawing ellipses freehand at all angles of inclination or you will find yourself depending on the guides instead of your eye. Remember that ellipse guides are just that – guides.

To draw an ellipse within a constructed circumscribing square, first find its centre using diagonals and then draw in the axis of rotation and a line at geometric right angles to it. Next choose the ellipse whose perimeter comes closest to touching the mid-point of each side of the square when the minor axis is aligned with the axis of rotation and the major axis is aligned with the line at right angles to it. You are unlikely, even with a full set of guides, to find the *exact* ellipse required in both size and inclination, so use the nearest one to it. If this still looks unsatisfactory, it is usually possible to move the guide slightly as you draw each half of the ellipse to approximate to the 'in-between' ellipses.

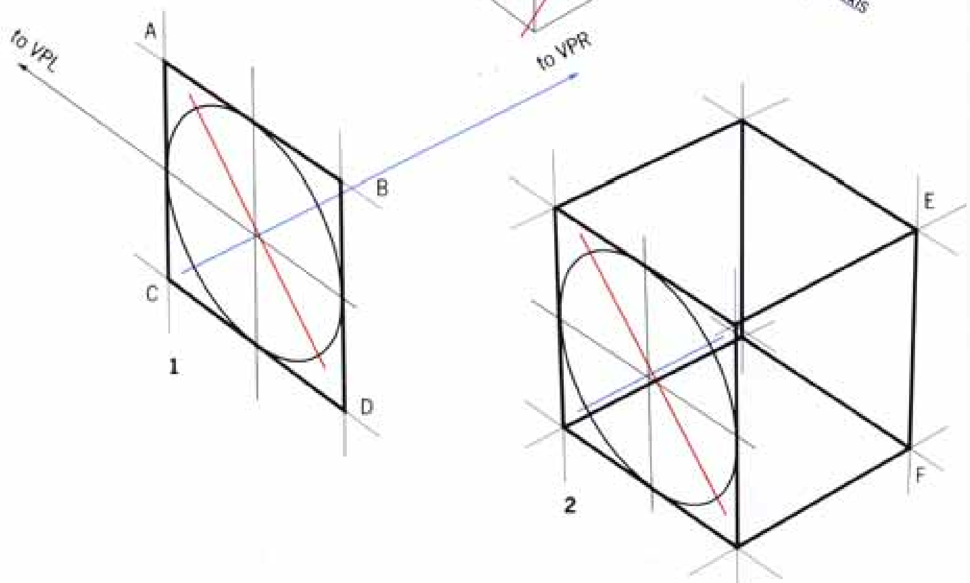
Remember also, that with a cylindrical object the ellipse nearest the viewer may be, say, a 35-degree ellipse but, as the cylinder recedes, the ellipses will come closer to the plan-view and you will find that a 40-degree and then a 45-degree will be a better fit in the circumscribing square. However, the further away the circumscribing square is from the viewer, the more distorted it will become and, equally, the more difficult it will be to accommodate an ellipse.

It is also useful to bear in mind, particularly for those with a limited selection of guides, that, if the product to be drawn is generally cylindrical, like a bottle or screwdriver, an ellipse template can be used to create the view without needing to start from scratch.



Changing the value of the ellipse

As a cylinder drawn in perspective recedes from the viewer, the value of the ellipse should change because we are seeing more of a plan-view of it.



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Constructing views from an ellipse guide

1. Lay in an ellipse of the right size with the minor axis at approximately the right inclination for your intended view; be sure to mark off the major and minor axes. Extend the minor axis in both directions and remember that this line must go to one of the vanishing points (VPR). Next drop two verticals, AC and BD, which are tangent to the perimeter and then draw a line that runs through the two tangent points and the centre of the ellipse; this line must go to the other vanishing point (VPL). Drop another vertical through the centre of the ellipse.

So far, little skill or judgement has been called for. The next step is to finish off the circumscribing square and to do this you will need to estimate the rate of diminishment (and therefore the proximity of the vanishing point) of the remaining two lines, AB and CD, with respect to the centre line already established. The two lines should recede to the vanishing point and each should be equidistant from the centre line as measured along a vertical drawn at any point. Be careful not to impose too great a degree of diminishment or the vanishing point will be too close and therefore distortion will be more likely. The resulting trapezoid should be a circumscribing square in perspective (given the offset centre anomaly we have chosen to ignore).

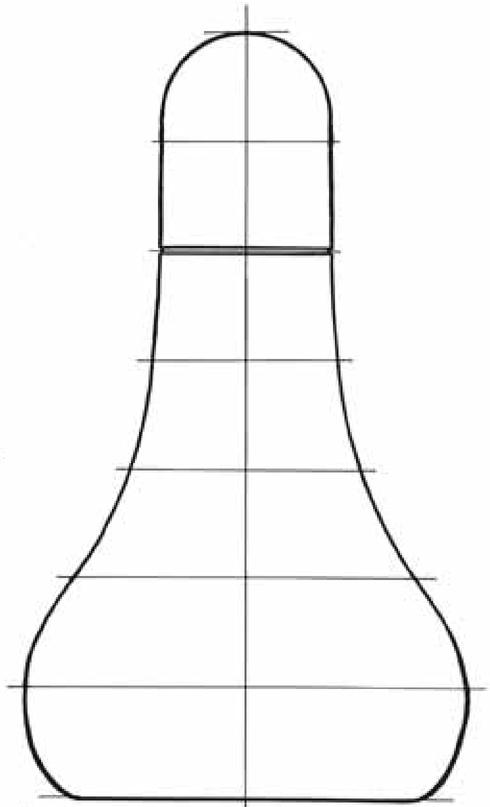
2. Next, use your judgement to draw in four perspective lines from each corner of the square to the other vanishing point, bearing in mind that they should all converge at the same rate. Start with the line closest to the axis of rotation and work outwards. Obviously, if the vanishing points are not 'off the board' and you want more accuracy, you can put in a horizon from the previously established vanishing point (VPL) and where this horizon line intersects the axis of rotation will be the second vanishing point (VPR). Next, you will have to estimate where to put the vertical (EF), which defines the second side of the cube, and this you will have to do completely by eye. Do it lightly at first and complete the rest of the cube, especially the unseen faces, *to give you the maximum visual information on which to base a decision. Look at the completed cube and decide whether it looks distorted. If it looks out of proportion, move the line to correct it and complete the rest of the cube again. With this method, remember that every step of the way, distortion can creep in, so it should only be treated as a quick guide.*

Right: Constructing a circular shape using an ellipse guide

1. Ellipse guides can also be very useful for building up views of complex circular shapes, such as bottles, from elevations, provided their axis of rotation is vertical. Simply take the elevation (or a single profile and centre-line) and draw horizontal section lines at regular intervals and at key changes of direction.

2. Because the bottle is small, we can use a single template. If, on the other hand, it was a much larger object, we would select one for the top, one for the middle and one for the base. Draw the ellipse which, with both axes aligned, has the profile passing through the point of intersection of the major axis and the perimeter of the ellipse. This will produce a skeletal view, and it only remains to connect up the resulting silhouette to complete the construction. Note that the silhouette and cross-section diverge away from one another as the bottle flares outwards and then in again at the base.

3. The whole process is equally useful but less easy when drawing the same product on its side. In this case it is obviously necessary to first construct, or sketch, the elevational view in perspective complete with vertical section lines. If you need to set up an accurate perspective, then this method probably isn't worth the trouble because you may as well construct all the necessary circumscribing squares. If, however, you are reasonably confident that you can do this at least partly by eye, it will save a lot of time. (You need to be able to divide a single line into equal parts with a regular rate of diminishment. The way I do this is to draw the line and mark off each end; then I estimate the mid-point in perspective to one side of the measured centre, divide the two halves in half again and so on). Next, draw a geometric right angle through each section centre to indicate the major axis. Then, offer up the ellipse guides until you find one which, when both axes are aligned, has its circumference passing through the point of intersection of the section-line and its corresponding vertical. Repeat this for each section to create a skeletal view and then join up the silhouette.



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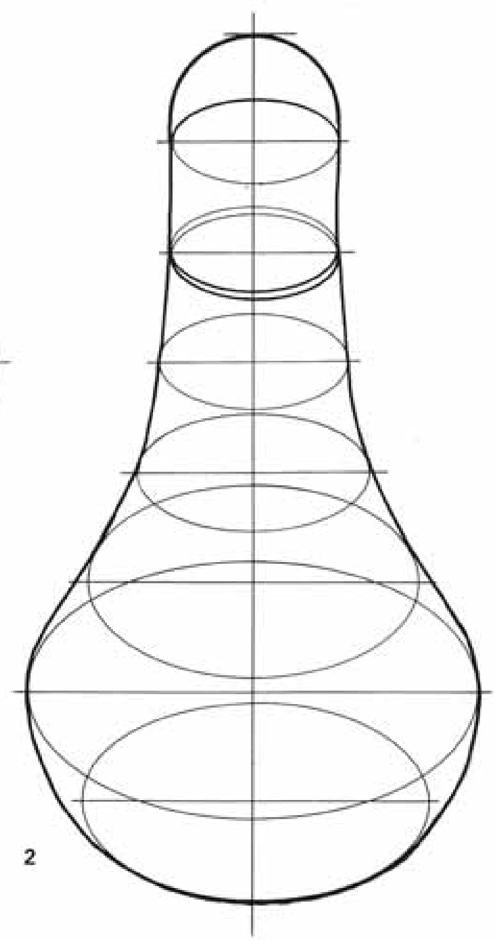

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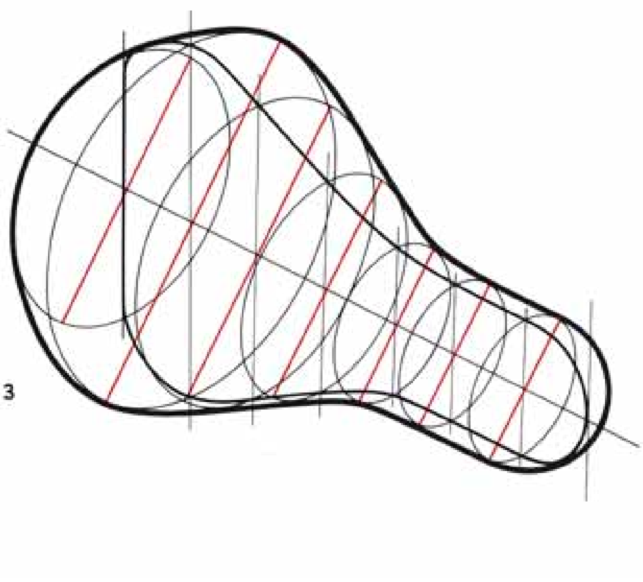

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